

## Towards a Higgs phase of gravity in string theory

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**ABSTRACT:** We consider a braneworld scenario with a black brane parallel to our brane, aiming towards a Higgs phase of gravity in string theory. The existence of the black brane spontaneously breaks the Lorentz symmetry on our brane but preserves the rotational and translational invariance. If all moduli are stabilized then this should lead to a Higgs phase of gravity. We investigate moduli stabilization by using a KKLT-type moduli potential in the context of type IIB warped flux compactification.

**KEYWORDS:** Spontaneous Symmetry Breaking, p-branes, Classical Theories of Gravity, dS vacua in string theory.

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## 1. Introduction

Dark energy and dark matter are two major mysteries in modern cosmology. Although more than 90% of our universe is filled with them, we do not know what they really are. In this situation it seems rather natural to ask whether we can modify gravity in the infrared (IR) to address those mysteries.

However, modifying gravity in the IR is not easy. For example, in massive gravity [1] and the DGP brane model [2], it is known that a scalar degree of freedom becomes strongly coupled at a rather low energy scale [3, 4]. If gravity is to be modified at and beyond the present Hubble distance then the scalar degree of freedom becomes strongly coupled and its dynamics is dominated by quantum corrections at and within  $\sim 1000km$ . This is similar to what happens to a massive gauge theory, but the situation is worse. In a massive gauge theory one can decouple the strongly coupled sector from the rest of the world by tuning the gauge coupling to a small value. On the other hand, in the massive gravity and the DGP model, the strength of interactions between the strongly coupled sector and the rest of the world is set by the Newton's constant and cannot be fine-tuned away. This means that, as soon as the strong coupling appears, gravitational phenomena cannot be properly described by those theories without knowledge about yet-unknown ultraviolet (UV) completion. Since this issue in the UV is originated from the properties in the IR, it

is not totally clear whether it is possible to UV complete those theories without affecting the IR behaviors.

In a gauge theory, it is the Higgs mechanism that gives a mass term to a gauge boson and modifies a force law in a theoretically controllable way. Thus, it is natural to apply the idea of the Higgs mechanism to gravity to modify its IR behavior. Since the symmetry relevant to gravity is diffeomorphism invariance, the Higgs mechanism for gravity should spontaneously break at least part of the diffeomorphism invariance. Note that in gravitational theories, Lorentz breaking inevitably implies breaking diffeomorphism invariance and, thus, a Higgs phase of gravity.

The simplest Higgs phase of gravity is the ghost condensation [5], including just one Nambu-Goldstone (NG) boson. Because of the simplicity and the universality of the low energy effective field theory (EFT), it is worthwhile investigating properties of the ghost condensation, whether or not it leads to interesting physical phenomena. Fortunately, it turns out that the physics in the simplest Higgs phase of gravity is extremely rich and interesting. They include IR modification of gravity [5], a new spin-dependent force [6], a qualitatively different picture of inflationary de Sitter phase [7, 8], effects of moving sources [9, 10], intriguing nonlinear dynamics [11, 12], properties of black holes [13–15], implications to galaxy rotation curves [16–18], dark energy models [19–21], stable violation of null energy condition [22], cosmological perturbations [23], other classical dynamics [24, 25], attempts towards UV completion [26, 27], and so on. In the ghost condensation the strength of interactions between the NG boson and the rest of the world is controlled by not only the Newton’s constant but also the scale of spontaneous Lorentz breaking  $\mathcal{M}$  in such a way that the interactions are completely turned off in the limit  $\mathcal{M}/M_{Pl} \rightarrow 0$ . Therefore, general relativity is safely recovered in this limit. In ref. [12] it was argued that the theory is compatible with all current experimental observations if  $\mathcal{M}$  is lower than  $\sim 100$  GeV. Therefore, the ghost condensation opens up a number of new avenues for attacking cosmological problems, including inflation, dark matter and dark energy.

While phenomenologies of the ghost condensation are still under investigation, it is certainly important to seek a possible UV completion.

To realize the ghost condensation without fine-tuning, we need to spontaneously break the 4-dimensional diffeomorphism invariance times a global shift symmetry down to the 3-dimensional spatial diffeomorphism invariance times an unbroken global shift symmetry, where the latter global shift is a combination of the former global shift and the time shift. However, it is generally believed that all symmetries in string theory are gauged. Therefore, it seems more plausible to obtain the ghost condensation as the neutral limit of the gauged version of the ghost condensation, i.e. the gauged ghost condensation [28]. To obtain the gauged ghost condensation from the ghost condensation we replace the global shift symmetry with a minimal gauge symmetry, i.e. U(1) gauge symmetry, so that no global symmetry is needed. The ghost condensation can be obtained from the gauged ghost condensation if we can fine-tune the gauge coupling to a sufficiently small value. Some of low energy properties of the gauged ghost condensation, including the bound  $\mathcal{M} \lesssim \text{Min}(10^{12} \text{ GeV}, g^2 10^{15} \text{ GeV})$  for  $g^2 > g_c^2$ , have been investigated in [28]. Here,  $g$  is the gauge coupling and  $g_c = \mathcal{M}^2/2M_{Pl}^2$ .

The purpose of this paper is, as a step towards a Higgs phase of gravity in string theory, to consider a braneworld scenario in which a black brane parallel to our brane breaks the Lorentz symmetry on our brane. If all moduli are stabilized then this leads to a Higgs phase of gravity in our world. From symmetry arguments, we expect that this is closely related to the gauged ghost condensation.

The rest of this paper is organized as follows. In section 2 we review the gauged ghost condensation by deriving its low energy EFT. This makes it clear that the structure of the EFT is determined solely by the symmetry breaking pattern. In other words, different setups should result in low energy EFTs with the same structure if the symmetry breaking pattern is the same. In section 3 we investigate the dispersion relation of the NG boson coupled with gravity and show that it exhibits Jeans-like instability if the gauge coupling in the EFT is smaller than a critical value. We also point out a number of similarities between the Jeans-like instability of the NG boson and the Gregory-Laflamme (GL) instability of black branes [29, 30]. Those similarities motivate a braneworld setup considered in section 5. In section 4 it is argued that, in (higher-dimensional) general relativity, a brane with codimension more than 2 should form a black brane if the brane is sufficiently thin. The string theory version of this statement is the correspondence principle for  $D$ -branes and black branes [31], which plays a central role in the braneworld setup. In section 4 we also see close connections between extremality of a black brane and the Lorentz symmetry along the world-volume of the black brane. Motivated by various considerations in the preceding sections, a braneworld setup in string theory is considered in section 5. Section 6 is devoted to a summary of this paper and discussions.

## 2. Gauged ghost condensation

The ghost condensation is the simplest Higgs phase of gravity in the sense that it contains only one Nambu-Goldstone boson. The gauged ghost condensation is obtained by gauging the global shift symmetry of the ghost condensation [28]. In this section we give an alternative derivation of the effective field theory (EFT) of the gauged ghost condensation based on a symmetry breaking pattern.

For the gauged ghost condensation, we assume the following symmetry breaking pattern.

- (i) The 4-dimensional diffeomorphism invariance and a  $U(1)$  gauge symmetry is spontaneously broken down. The residual symmetries are the 3-dimensional spatial diffeomorphism invariance, the time reversal symmetry and a  $U(1)$  gauge symmetry. The residual  $U(1)$  is a combination of the original  $U(1)$  and the time reparameterization.
- (ii) The background spacetime metric is maximally symmetric, either Minkowski or de Sitter.

Our strategy here is to write down the most general action invariant under the residual symmetries. This gives the effective action in the unitary gauge. After that, the action in a general gauge is obtained by gauge transformation. The gauge parameter associated

with the broken symmetry actually appears in the action and is identified as the NG boson associated with the spontaneous symmetry breaking.

For simplicity let us consider a Minkowski background plus perturbation:  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ . We also consider a U(1) gauge field  $a_\mu$ . The general infinitesimal diffeomorphism and U(1) gauge transformations are  $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$  and  $\delta a_\mu = \partial_\mu \chi$ , where the 4-vector  $\xi^\mu$  and the scalar  $\chi$  represent gauge degrees of freedom. The residual 3-dimensional spatial diffeomorphism is given by

$$\xi^0 = 0, \quad \xi^i = \xi^i(t, \vec{x}), \quad \chi = 0, \quad (2.1)$$

and transforms the metric perturbation and the U(1) gauge field as

$$\delta h_{00} = 0, \quad \delta h_{0i} = \partial_0 \xi_i, \quad \delta h_{ij} = \vec{\nabla}_i \xi_j + \vec{\nabla}_j \xi_i, \quad \delta a_0 = \delta a_i = 0. \quad (2.2)$$

The residual U(1) gauge symmetry is characterized by

$$\xi^0 (= -\xi_0) = -\frac{g}{\mathcal{M}} \chi(t, \vec{x}), \quad \xi^i = 0, \quad \chi = \chi(t, \vec{x}), \quad (2.3)$$

and transforms the fields as

$$\delta h_{00} = \frac{2g}{\mathcal{M}} \partial_0 \chi, \quad \delta h_{0i} = \frac{g}{\mathcal{M}} \vec{\nabla}_i \chi, \quad \delta h_{ij} = 0, \quad \delta a_0 = \partial_0 \chi, \quad \delta a_i = \vec{\nabla}_i \chi, \quad (2.4)$$

where  $\mathcal{M}$  is the scale of the spontaneous Lorentz breaking and  $g$  is the gauge coupling. On the other hand, the choice

$$\xi^0 (= -\xi_0) = \pi(t, \vec{x}), \quad \xi^i = 0, \quad \chi = 0 \quad (2.5)$$

corresponds to the broken symmetry and the fields are transformed as

$$\delta h_{00} = -2\partial_0 \pi, \quad \delta h_{0i} = -\vec{\nabla}_i \pi, \quad \delta h_{ij} = 0, \quad \delta a_0 = \delta a_i = 0. \quad (2.6)$$

Now let us seek terms invariant under the both residual gauge transformations (2.2) and (2.4). They must begin at quadratic order since we assumed that the Minkowski background is a solution to the equation of motion. The leading term (without derivatives acted on the metric perturbations) is  $\int d\vec{x}^3 dt \tilde{h}_{00}^2$ , where

$$\tilde{h}_{00} \equiv h_{00} - \frac{2g}{\mathcal{M}} a_0. \quad (2.7)$$

This is indeed invariant under both residual gauge transformations (2.2) and (2.4). From this term, we can obtain the corresponding term in the effective action for the NG boson  $\pi$ . Since  $\tilde{h}_{00} \rightarrow \tilde{h}_{00} - 2\partial_0 \pi$  under the broken symmetry transformation (2.6), by promoting  $\pi$  to a physical degree of freedom we obtain the term  $\int d\vec{x}^3 dt (\tilde{h}_{00} - 2\partial_0 \pi)^2$ . This includes a time kinetic term for  $\pi$  as well as mixing terms. At this point we wonder if we can get the usual space kinetic term  $(\vec{\nabla} \pi)^2$  or not. The only possibility would be from  $(h_{0i})^2$  since  $h_{0i} \rightarrow h_{0i} - \vec{\nabla}_i \pi$  under the broken symmetry transformation (2.6). However, this term is not invariant under either (2.2) or (2.4), and, thus, cannot enter the effective action. Actually, by acting derivatives on the metric components, we can find combinations manifestly invariant under

the spatial diffeomorphism (2.2). They are made of the geometrical quantity called extrinsic curvature. The extrinsic curvature  $K_{ij}$  of a constant time surface is  $K_{ij} = (\partial_0 h_{ij} - \vec{\nabla}_i h_{0j} - \vec{\nabla}_j h_{0i})/2$  in the linear order and transforms as a tensor under the spatial diffeomorphism. Indeed, it is invariant under the spatial diffeomorphism (2.2) since the background value of the extrinsic curvature vanishes. Although it is not invariant under the residual U(1) gauge transformation (2.4), it is possible to compensate the transformation of  $K_{ij}$  with that of derivatives of the U(1) gauge field. Indeed, the combination

$$\tilde{K}_{ij} \equiv K_{ij} + \frac{g}{\mathcal{M}} \vec{\nabla}_{(i} a_{j)} \quad (2.8)$$

is invariant under both (2.2) and (2.4), where  $\vec{\nabla}_{(i} a_{j)} \equiv (\vec{\nabla}_i a_j + \vec{\nabla}_j a_i)/2$ . Thus,  $\int d\vec{x}^3 dt \tilde{K}^2$  and  $\int d\vec{x}^3 dt \tilde{K}^{ij} \tilde{K}_{ij}$  can be used in the action. Since  $\tilde{K}_{ij} \rightarrow \tilde{K}_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi$  under the broken symmetry (2.6), we obtain  $\int d\vec{x}^3 dt (\tilde{K} + \vec{\nabla}^2 \pi)^2$  and  $\int d\vec{x}^3 dt (\tilde{K}^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi)(\tilde{K}_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi)$ . The field strength  $F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$  is also invariant under both (2.2) and (2.4). Thus,  $\int d\vec{x}^3 dt F_0^i F_{0i}$  and  $\int d\vec{x}^3 dt F^{ij} F_{ij}$  can enter the action. These do not generate terms involving the NG boson. Note that terms like  $\int d\vec{x}^3 dt \tilde{K} \tilde{h}_{00}$  are forbidden by the time reversal symmetry.

Combining these terms, we obtain the effective action  $\int d\vec{x}^3 dt L$ , where

$$\begin{aligned} L = & \frac{\mathcal{M}^4}{2} \left( \partial_0 \pi + \frac{g}{\mathcal{M}} a_0 - \frac{1}{2} h_{00} \right)^2 - \frac{\alpha_1 \mathcal{M}^2}{2} \left( \vec{\nabla}^2 \pi + \frac{g}{\mathcal{M}} \vec{\nabla}^i a_i + K \right)^2 \\ & - \frac{\alpha_2 \mathcal{M}^2}{2} \left( \vec{\nabla}^i \vec{\nabla}^j \pi + \frac{g}{\mathcal{M}} \vec{\nabla}^{(i} a^{j)} + K^{ij} \right) \left( \vec{\nabla}_i \vec{\nabla}_j \pi + \frac{g}{\mathcal{M}} \vec{\nabla}_{(i} a_{j)} + K_{ij} \right) \\ & + \frac{1}{2} F_0^i F_{0i} - \frac{\gamma_1}{4} F^{ij} F_{ij} + \dots \end{aligned} \quad (2.9)$$

Here,  $\alpha_{1,2}$  and  $\gamma_1$  are dimensionless constants of order unity and we have normalized  $\pi$ ,  $a_\mu$  and  $g$  so that the coefficients of the first and the fourth terms become  $\mathcal{M}^4/2$  and  $1/2$ , respectively.

In deriving the effective action (2.9), all we needed was the symmetry breaking pattern. As intended, this action agrees with the effective action obtained in ref. [28] by simply gauging the shift symmetry in the ghost condensation.

This action has the following symmetry:

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad a_\mu \rightarrow a_\mu + \partial_\mu \chi, \quad \pi \rightarrow \pi + \xi_0 - \frac{g}{\mathcal{M}} \chi, \quad (2.10)$$

where the gauge parameters  $\xi^\mu$  and  $\chi$  depend on  $t$  and  $\vec{x}$ . By using  $\chi$ , we can set  $a_0 = 0$  and obtain the effective action

$$\begin{aligned} L = & \frac{\mathcal{M}^4}{2} \left( \partial_0 \pi - \frac{1}{2} h_{00} \right)^2 - \frac{\alpha_1 \mathcal{M}^2}{2} \left( \vec{\nabla}^2 \pi + \frac{g}{\mathcal{M}} \vec{\nabla}^i a_i + K \right)^2 \\ & - \frac{\alpha_2 \mathcal{M}^2}{2} \left( \vec{\nabla}^i \vec{\nabla}^j \pi + \frac{g}{\mathcal{M}} \vec{\nabla}^{(i} a^{j)} + K^{ij} \right) \left( \vec{\nabla}_i \vec{\nabla}_j \pi + \frac{g}{\mathcal{M}} \vec{\nabla}_{(i} a_{j)} + K_{ij} \right) \\ & + \frac{1}{2} \partial_0 a^i \partial_0 a_i - \frac{\gamma_1}{4} F^{ij} F_{ij} + \dots \end{aligned} \quad (2.11)$$

This action has the following symmetry:

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad a_i \rightarrow a_i + \vec{\nabla}_i \chi^{(3)}, \quad \pi \rightarrow \pi + \xi_0 - \frac{g}{\mathcal{M}} \chi^{(3)}, \quad (2.12)$$

where  $\chi^{(3)}$  depends on  $\vec{x}$  but is independent of  $t$ .

If the gauge coupling in the EFT is small enough then this reduces to the ghost condensation [5]:

$$L_{g \rightarrow 0} = \frac{\mathcal{M}^4}{2} \left( \partial_0 \pi - \frac{1}{2} h_{00} \right)^2 - \frac{\alpha_1 \mathcal{M}^2}{2} \left( \vec{\nabla}^2 \pi + K \right)^2 - \frac{\alpha_2 \mathcal{M}^2}{2} \left( \vec{\nabla}^i \vec{\nabla}^j \pi + K^{ij} \right) \left( \vec{\nabla}_i \vec{\nabla}_j \pi + K_{ij} \right) + \dots, \quad (2.13)$$

This action includes leading terms of the quadratic order only. Actually, it is possible to show that nonlinear terms are irrelevant at low energies. To begin with, suppose that the energy is scaled as  $E \mapsto sE$ . Then, the time interval  $dt$  scales as  $dt \mapsto s^{-1}dt$ . To determine the scaling rule for the spatial interval  $d\vec{x}$  and the NG boson  $\pi$ , we demand that the leading action  $\int d\vec{x}^3 dt L_{g \rightarrow 0}$  with  $h_{\mu\nu} = 0$  be invariant under the scaling. The results are summarized as

$$\begin{aligned} E &\mapsto sE, \\ dt &\mapsto s^{-1}dt, \\ d\vec{x} &\mapsto s^{-1/2}d\vec{x}, \\ \pi &\mapsto s^{1/4}\pi. \end{aligned} \quad (2.14)$$

Note that the scaling dimension of  $\pi$  is not equal to the mass dimension 1 but is 1/4 [5]. By using this scaling, it is easy to identify the scaling dimensions of any nonlinear operators. The leading nonlinear operator

$$\int d\vec{x}^3 dt \mathcal{M}^4 \dot{\pi} (\vec{\nabla} \pi)^2 \quad (2.15)$$

scales as  $s^{1/4}$  and is irrelevant at low energies. All other nonlinear operators have scaling dimension 1/4 or higher and, thus, are irrelevant. Therefore, those nonlinear operators become less and less important as energies and momenta become low compared with the scale  $\mathcal{M}$ . Low energy dynamics of the NG boson is, thus, well described by the effective action.

Going back to the action (2.11) for a non-vanishing  $g$ , the above power-counting analysis shows that at low energies and momenta, all nonlinear operators made of  $\pi$  and its derivatives are irrelevant compared with the first three terms. On the other hand, the fourth and fifth terms in (2.11) give leading kinetic terms for  $a_i$ . Therefore, the action (2.11) well describes the low energy dynamics of the system.

In summary, (2.11) is the low energy effective of the gauged ghost condensation characterized by the symmetry breaking pattern in the beginning of this section. The structure of the effective action is universal since it is determined solely by the symmetry breaking

pattern. Different setups of Higgs phase of gravity should result in low energy EFTs of the same structure if their symmetry breaking patterns are the same. If the gauge coupling  $g$  is small enough then the effective action is reduced to (2.13) and agrees with that of the ghost condensation.

### 3. Jeans-like instability and Gregory-Laflamme instability

The EFT (2.11) should be coupled to Einstein gravity. As we shall see below, qualitative behavior of the coupled system depends on whether the gauge coupling  $g$  is larger than a critical value  $g_c$  or smaller [28]. For  $g^2 > g_c^2$ , the linear perturbation around Minkowski background is stable. On the other hand, for  $g^2 < g_c^2$ , the coupled system exhibits a Jeans-like instability for long wavelength modes as in the ghost condensation [5]. In the end of this section we shall point out similarities between this Jeans-like instability and the Gregory-Laflamme (GL) instability [29, 30] of black branes. This similarity is what will motivate us to consider a scenario for a Higgs phase of gravity in string theory explained in section 5.

Let us now derive the dispersion relation for the NG boson described by (2.11). Since the 3-dimensional spatial diffeomorphism invariance is preserved, linear perturbations around the background can be decomposed into scalar, vector and tensor type, according to their transformation properties under the spatial diffeomorphism. In this section we consider scalar-type perturbation.

For scalar-type perturbation,  $a_i$  is written as  $a_i = \vec{\nabla}_i a_L$ , where  $a_L$  depends on  $t$  and  $\vec{x}$ . By setting  $\alpha_2 = 0$  for simplicity, the action (2.11) then reduces to

$$L = \frac{\mathcal{M}^4}{2} \left( \partial_0 \pi - \frac{1}{2} h_{00} \right)^2 - \frac{\alpha_1 \mathcal{M}^2}{2} \left( \vec{\nabla}^2 \pi + \frac{g}{\mathcal{M}} \vec{\nabla}^2 a_L + K \right)^2 + \frac{1}{2} (\partial_0 \vec{\nabla} a_L)^2. \quad (3.1)$$

Thus, the equations of motion derived from the total action  $\int d\vec{x}^3 dt (L_{EH} + L)$  are

$$\begin{aligned} M_{\text{Pl}}^2 G_{00} + \mathcal{M}^4 \left( \partial_0 \pi - \frac{1}{2} h_{00} \right) &= 0, \\ M_{\text{Pl}}^2 G_{0i} - \alpha_1 \mathcal{M}^2 \vec{\nabla}_i \left( \vec{\nabla}^2 \pi + \frac{g}{\mathcal{M}} \vec{\nabla}^2 a_L + K \right) &= 0, \\ M_{\text{Pl}}^2 G_{ij} - \alpha_1 \mathcal{M}^2 \partial_0 \left( \vec{\nabla}^2 \pi + \frac{g}{\mathcal{M}} \vec{\nabla}^2 a_L + K \right) \delta_{ij} &= 0, \\ \partial_0^2 a_L - \alpha_1 g \mathcal{M} \left( \vec{\nabla}^2 \pi + \frac{g}{\mathcal{M}} \vec{\nabla}^2 a_L + K \right) &= 0, \\ \partial_0 \left( \partial_0 \pi - \frac{1}{2} h_{00} \right) + \frac{\alpha_1}{\mathcal{M}^2} \vec{\nabla}^2 \left( \vec{\nabla}^2 \pi + \frac{g}{\mathcal{M}} \vec{\nabla}^2 a_L + K \right) &= 0. \end{aligned} \quad (3.2)$$

In the longitudinal gauge

$$ds_4^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)d\vec{x}^2, \quad (3.3)$$



the Einstein tensor and the extrinsic curvature are

$$\begin{aligned}
 G_{00} &= 2\vec{\nabla}^2\Psi, \\
 G_{0i} &= 2\partial_0\vec{\nabla}_i\Psi, \\
 G_{ij} &= 2\left[\partial_0^2\Psi + \frac{1}{3}\vec{\nabla}^2(\Phi - \Psi)\right]\delta_{ij} - \left(\vec{\nabla}_i\vec{\nabla}_j - \frac{1}{3}\vec{\nabla}^2\delta_{ij}\right)(\Phi - \Psi), \\
 K_{ij} &= -\partial_0\Psi\delta_{ij}.
 \end{aligned} \tag{3.4}$$

Thus, the traceless part of the third equation of motion implies  $\Phi = \Psi$ . The rest of the equations of motion become

$$\begin{aligned}
 2M_{\text{Pl}}^2\vec{\nabla}^2\Phi + \mathcal{M}^4(\partial_0\pi + \Phi) &= 0, \\
 2M_{\text{Pl}}^2\partial_0\vec{\nabla}_i\Phi - \alpha_1\mathcal{M}^2\vec{\nabla}_i\left(\vec{\nabla}^2\pi + \frac{g}{\mathcal{M}}\vec{\nabla}^2a_L - 3\partial_0\Phi\right) &= 0, \\
 2M_{\text{Pl}}^2\partial_0^2\Phi - \alpha_1\mathcal{M}^2\partial_0\left(\vec{\nabla}^2\pi + \frac{g}{\mathcal{M}}\vec{\nabla}^2a_L - 3\partial_0\Phi\right) &= 0, \\
 \partial_0^2a_L - \alpha_1g\mathcal{M}\left(\vec{\nabla}^2\pi + \frac{g}{\mathcal{M}}\vec{\nabla}^2a_L - 3\partial_0\Phi\right) &= 0, \\
 \partial_0(\partial_0\pi + \Phi) + \frac{\alpha_1}{\mathcal{M}^2}\vec{\nabla}^2\left(\vec{\nabla}^2\pi + \frac{g}{\mathcal{M}}\vec{\nabla}^2a_L - 3\partial_0\Phi\right) &= 0.
 \end{aligned} \tag{3.5}$$

The second and third equations imply that

$$\partial_0\Phi = \frac{\alpha_1g_c^2}{1 + 3\alpha_1g_c^2}\left(\vec{\nabla}^2\pi + \frac{g}{\mathcal{M}}\vec{\nabla}^2a_L\right), \tag{3.6}$$

where

$$g_c^2 = \frac{\mathcal{M}^2}{2M_{\text{Pl}}^2}. \tag{3.7}$$

The physical meaning of  $g_c$  will be clarified soon. By substituting this to the fifth equation, we obtain

$$\partial_0^2\pi + \frac{\alpha_1}{1 + 3\alpha_1g_c^2}\left(g_c^2 + \frac{\vec{\nabla}^2}{\mathcal{M}^2}\right)\left(\vec{\nabla}^2\pi + \frac{g}{\mathcal{M}}\vec{\nabla}^2a_L\right) = 0. \tag{3.8}$$

Finally, by acting the operator  $[\partial_0^2 - \alpha_1g^2\vec{\nabla}^2/(1 + 3\alpha_1g_c^2)]$  on this equation and using the fourth equation (with the substitution of (3.6)), we obtain

$$\partial_0^2\left[\partial_0^2 + \frac{\alpha_1}{1 + 3\alpha_1g_c^2}\left(-g^2 + g_c^2 + \frac{\vec{\nabla}^2}{\mathcal{M}^2}\right)\vec{\nabla}^2\right]\pi = 0. \tag{3.9}$$

Thus, by neglecting  $3\alpha_1g_c^2 (\ll 1)$  in the denominator, this equation implies the following dispersion relation:

$$\omega^2 = \alpha_1(g^2 - g_c^2)\vec{k}^2 + \frac{\alpha_1}{\mathcal{M}^2}\vec{k}^4, \tag{3.10}$$

where  $g_c$  is defined by (3.7). For the stability of modes with high momenta, we assume that  $\alpha_1 > 0$ . When  $g^2 \ll g_c^2$ , the gauged ghost condensation reduces to the ghost condensation and the dispersion relation becomes

$$\omega^2 = -\frac{\alpha_1\mathcal{M}^2}{2M_{\text{Pl}}^2}\vec{k}^2 + \frac{\alpha_1}{\mathcal{M}^2}\vec{k}^4 \quad \text{for } g^2 \ll g_c^2. \tag{3.11}$$

From the dispersion relation (3.10) it is easy to see that the NG boson coupled with the U(1) field and gravity is stable if  $g^2 > g_c^2$ . On the other hand, for  $g^2 < g_c^2$ , modes with long wavelength are unstable. For a usual fluid with the background energy density  $\rho_0$ , the dispersion relation is  $\omega^2 = c_s^2 \mathbf{k}^2 - \omega_J^2$ , where  $c_s$  is the sound speed and  $\omega_J^2 = 4\pi G_N \rho_0$ , and long-scale modes with  $c_s^2 \mathbf{k}^2 < \omega_J^2$  have instability called Jeans instability. The Jeans instability is originated from the attractive nature of gravity, and is an important instability rather than catastrophe since it contributes to the structure formation in the universe. The long-scale instability indicated by the dispersion relation (3.10) for  $g^2 < g_c^2$  is an analog of the Jeans instability. In the limit  $g^2 \ll g_c^2$ , from (3.11) one can see that the length and time scales characterizing the most unstable mode are

$$r_c \simeq \frac{M_{\text{Pl}}}{\mathcal{M}^2}, \quad t_c \simeq \frac{M_{\text{Pl}}^2}{\mathcal{M}^3} \quad \text{for } g^2 \ll g_c^2, \quad (3.12)$$

and are much longer than  $1/\mathcal{M}$ . For larger  $g^2$  ( $< g_c^2$ ), the length and time scales are even longer.

Thus, the physical meaning of  $g_c$  is the critical value of the gauge coupling below which the NG boson exhibits the Jeans-like instability. For small gauge coupling  $g^2 < g_c^2$ , the attractive nature of gravity dominates over the repulsive U(1) gauge force. This is the reason for the Jeans-like instability, which reflects the attractive nature of gravity. On the other hand, for large gauge coupling  $g^2 > g_c^2$ , the repulsive nature of the U(1) gauge force dominates over the attractive nature of gravity and the Jeans-like instability disappears. Of course, even for  $g^2 < g_c^2$ , the Jeans-like instability disappears in the expanding universe if the Hubble expansion rate is large enough.

Gregory-Laflamme (GL) instability [29, 30] of black brane solutions is a classical instability in which the mass of the black brane tends to clump non-uniformly. Intriguingly, as we shall point out in the following, there are a couple of similarities to the Jeans-like instability of the NG boson explained above.

First, both instabilities are for long wavelength modes. Modes with wavelength shorter than a critical length are stable in both cases. Indeed, the dispersion relation for the GL instability is qualitatively the same as (3.10) with  $g^2 < g_c^2$  as one can see from figure 1 of [29] and figure 6 of [30]. Note that the momentum  $\mu$  along the world-volume of the black brane must be replaced by  $\sqrt{k^2}$  and the growth rate  $\Omega$  must be replaced by  $\sqrt{-\omega^2}$  in order to compare these two dispersion relations.

Second, both instabilities disappear when the Lorentz symmetry is recovered. The recovery of the Lorentz symmetry corresponds to the limit  $\mathcal{M}/M_{\text{Pl}} \rightarrow 0$  in the above EFT coupled to gravity, and in this limit the Jeans-like instability indeed disappears ( $r_c \rightarrow \infty$  and  $t_c \rightarrow \infty$ ) even for  $g^2 \ll g_c^2$ . On the other hand, as we shall see in section 4 and appendix A, a non-extremal black brane tends to break the Lorentz symmetry along its world-volume: a non-extremal black brane has a preferred frame in which it is at rest, and a Lorentz boost even in the direction parallel to its world-volume does not preserve the form of the metric. However, as we shall see explicitly for a four-parameter family of black  $p$ -brane solutions, when the Lorentz symmetry is recovered, a black brane becomes extremal and BPS-saturated. It is thought that extremal black  $p$ -branes are stable and

do not exhibit the GL instability [32]. Hence, the GL instability also disappears when the Lorentz symmetry along the world-volume of the black brane is recovered.

Having seen these similarities, it is perhaps tempting to speculate that the GL instability of a non-extremal black brane might be interpreted as the Jeans-like instability of the NG boson associated with the spontaneous Lorentz breaking along the world-volume of the black brane. If this is true in some sense then we should expect close relations between the NG boson  $\pi$  in the EFT and the mass density of the black brane since they are the degrees of freedom exhibiting the instabilities. Similarly, the spatial components  $a_i$  of the U(1) gauge field should be related to the velocity field of the black brane mass density. To confirm these expectations is far beyond the scope of this paper, but it is certainly worthwhile investigating these expected relations by detailed analysis.

In summary, if the gauge coupling in the gauged ghost condensation is smaller than a critical value then the NG boson coupled with gravity exhibits a Jeans-like instability for long wavelength modes as in the ghost condensation [5]. We have pointed out similarities between the Jeans-like instability of the NG boson and the Gregory-Laflamme (GL) instability [29, 30] of black branes and speculated possible correspondence between these two instabilities.

#### 4. Non-extremal black brane and Lorentz breaking

As already mentioned in the previous section, a black brane can break the Lorentz symmetry along its world-volume. This is due to different radial dependence of the time-time and space-space components of the metric. Hence, it is tempting to seek a setup in which a black brane leads to a Higgs phase of gravity in our 4-dimensional world. For this purpose, it is necessary to embed our 4-dimensional world parallel to the world-volume of the black brane. This inevitably leads us to consider braneworld scenarios, in which our 4-dimensional world is supposed to be a brane in a higher dimensional bulk spacetime.

In braneworld scenarios, branes other than our world can be included and some of them might be black branes. Actually, in (higher-dimensional) general relativity, branes with more than two codimensions tend to form black branes in the limit where their thickness becomes sufficiently small. On the other hand, in the case of codimension-1, the dynamics of a brane coupled with higher dimensional gravity is consistently described by Israel's junction condition [33] even in the thin brane limit. The thin brane limit of codimension-2 objects is more subtle. If the higher-dimensional geometry surrounding a codimension-2 brane is axisymmetric and if the brane energy momentum is that of vacuum energy, or tension, then gravity around the brane in the thin brane limit is consistently described by a deficit angle. However, if either of the two conditions (the axisymmetry in the bulk and the brane energy momentum of vacuum energy type) is violated then the description breaks down. For codimensions more than two, there is no well-defined thin brane limit in (higher-dimensional) general relativity and a brane tends to form a black brane.

As already stated, it often happens that a black brane breaks the Lorentz symmetry along its world-volume. This is essentially because a black brane has a preferred frame in

which it is at rest and a Lorentz boost even in the direction parallel to its world-volume does not preserve the form of the metric. In other words, the time-time component and the space-space components of the metric depend differently on the radial coordinate.

In appendix A we consider a simple example in which we can explicitly see the Lorentz breaking along the world-volume of a black brane. We investigate a four-parameter family of  $p$ -brane solutions ( $p = 3, 4, 5, 6$ ) in type II A/B supergravity. It is shown that the regularity of the horizon sets two independent constraints on the parameters of the solutions and, thus, reduces the four-parameter family of solutions to a two-parameter family. As a result, a regular non-extremal black  $p$ -brane in the family of solutions always breaks the  $(p + 1)$ -dimensional Lorentz symmetry along its world-volume. On the other hand, the  $p$ -dimensional spatial rotational invariance and the  $(p + 1)$ -dimensional translational invariance are preserved. Intriguingly, the Lorentz symmetry is restored if and only if the black brane becomes extremal and BPS-saturated.

We expect that this connection between the non-extremality and the Lorentz breaking should hold in more general situations. Thus, a non-extremal black brane may be used as a source of spontaneous Lorentz breaking and may lead to a Higgs phase of gravity.

## 5. Setup in string theory

M/string theory has been considered as a strong candidate for a unified theory of fundamental physics. Its mathematical consistency and beauty have been attracting interest of many physicists. On the other hand, one of its drawbacks is lack of direct experimental or observational evidence of such a structure at high energies. Having this situation, it seems rather natural to turn our eyes to cosmology and look for cosmological implications of M/string theory since the universe is supposed to have experienced a high energy epoch at its early stage.

In order to realize realistic cosmological scenarios in string theory, one of the most important issues is the moduli stabilization. In the context of the type IIB superstring theory, Kachru, Kallosh, Linde and Trivedi (KKLT) [34] stabilized all moduli by using various fluxes and non-perturbative corrections to the moduli potential. Thus, this is a good starting point for cosmology in string theory.

In the KKLT setup anti- $D3$ -branes play an essential role. Inclusion of anti- $D3$ -branes at the bottom of a warped throat uplifts stable AdS vacua with negative cosmological constant to meta-stable de Sitter or Minkowski vacua with positive or zero cosmological constant in a theoretically controllable way. Without anti- $D3$ -branes or other alternative sources such as a  $D7$ -brane with non-zero flux inside its world-volume [35, 36], we would end up with a negative cosmological constant, which is inconsistent with observations.

In this section we consider a situation in which the anti- $D3$ -branes at the bottom of a warped throat may be described by a black brane and spontaneously break a part of the Lorentz symmetry along its world volume.

Before going into the setup, however, let us list what we have learned from various considerations in the previous sections.

- (a) The structure of the low energy effective field theory (EFT) of gauged ghost condensation is determined solely by the symmetry breaking pattern. Thus, all we have to do is to find a setup realizing the same symmetry breaking pattern.
- (b) Similarities between the Jeans-like instability of the Nambu-Goldstone (NG) boson and the Gregory-Laflamme (GL) instability of black branes suggest using a black brane.
- (c) A non-extremal black brane tends to spontaneously break the Lorentz symmetry along its world-volume.
- (d) In order for a black brane to break the Lorentz symmetry in our world, our world must be parallel to the world-volume of the black brane. This inevitably leads us to consider a braneworld scenario.
- (e) In (higher-dimensional) general relativity, branes with codimensions more than two tend to form black branes when the brane thickness is sufficiently small. The string theory version of this statement is the correspondence principle for stringy objects and black objects [31].

These suggest a braneworld scenario with a black brane parallel to our brane.<sup>1</sup> Consistent braneworld scenarios in string theory can be constructed in the KKLT setup of warped flux compactification, which realizes de Sitter vacua in string theory. Thus, we shall start with the warped flux compactification and seek a condition under which the correspondence principle [31] states that a black brane should form. In the setup, the world-volume of the black brane shall be parallel to a brane representing our world.

We consider the KKLT setup of Type IIB compactification with NS-NS and R-R fluxes. One begins with a warped throat generated by fluxes [39] and glues it to a bulk Calabi-Yau 3-fold to have compact extra dimensions [40]. The volume of the internal space is stabilized by non-perturbative effects such as  $D$ -instantons [41]. Since the 4-dimensional cosmological constant for this supersymmetric configuration is negative, KKLT [34] adds anti- $D3$ -branes at the tip of the warped throat and explicitly breaks supersymmetry to uplift the AdS vacua to meta-stable de Sitter vacua. Even if anti- $D3$ -branes are initially placed at some other places in the internal space, they feel attractive force towards the tip of the throat because of the non-BPS nature of the configuration [42]. Hence, the anti- $D3$ -branes fall towards the tip and finally settle there because of the Hubble friction due to expansion of the 4-dimensional universe [43].

The geometry deep inside the warped throat is approximated by the Klebanov-Strassler (KS) solution [39]. The bottom of the KS throat is actually not a point but has the topology  $\mathbf{R}^4 \times S^3$ , where  $\mathbf{R}^4$  represents the 4-dimensional universe and the radius of the  $S^3$  is of order  $\sqrt{g_s M} l_s$ , where  $M$  is an integer representing a quantized flux around the  $S^3$  [44]. Thus,

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<sup>1</sup>Similar attempts in the context of higher-dimensional general relativity require violation of the null energy condition or inclusion of naked singularities other than branes with codimension one [37, 38]. Our setup includes branes with higher codimensions as well as non-perturbative effects, which were not included in those attempts.

if  $g_s M \gg 1$  then the curvature of the geometry is everywhere small and the supergravity approximation is justified. Throughout this paper we assume this condition. In order for the string perturbative expansion to be valid, we require  $g_s \lesssim 1$  as well. In summary, we assume that

$$g_s M \gg 1, \quad g_s \lesssim 1. \tag{5.1}$$

Note that this configuration is only meta-stable. The anti- $D3$ -branes can annihilate with the  $D3$  charge induced by the background fluxes via quantum tunneling. Therefore, the configuration lasts for only a finite duration although the lifetime can be made longer than the age of the universe.

### 5.1 Moduli stabilization

For this setup to work it is important to make sure that the KKLT type moduli stabilization is valid. We have to stabilize both complex structure moduli and Kähler moduli in the system. Since all complex structure moduli are stabilized by fluxes, in this subsection we consider stabilizing Kähler moduli. In the following, for simplicity we shall consider only one Kähler modulus. We make a comment on the possibility of having many Kähler moduli in the end of subsection 5.3.

The supersymmetric contribution to the potential for the volume modulus  $\sigma$  combined with the axion  $\alpha$  is specified by the Kähler and super potentials of the form

$$K = -3 \log(T + \bar{T}), \quad W = W_0 + Ae^{-aT}, \tag{5.2}$$

where  $T = \sigma + i\alpha$ . As in the original KKLT setup, we uplift the AdS vacua to de Sitter vacua by introducing anti- $D3$ -branes, which adds a non-supersymmetric contribution of the form

$$\delta V = \frac{D}{(T + \bar{T})^2}, \quad D = \frac{2a_0^4 T_3 \bar{N}_3}{\pi^2 g_s^4}, \tag{5.3}$$

where

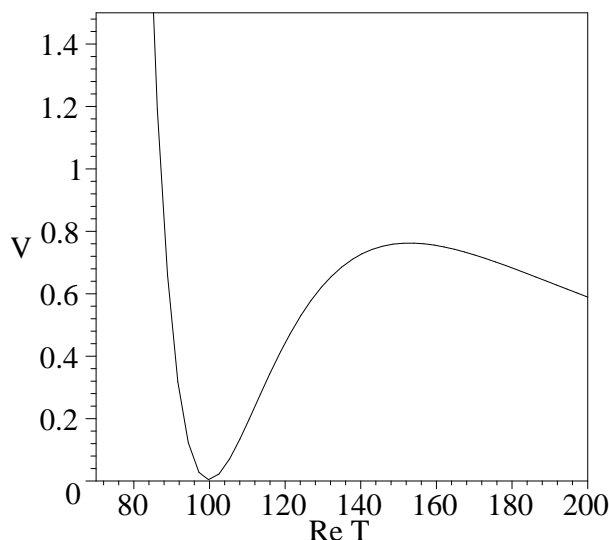
$$a_0 \simeq \exp\left(-\frac{2\pi K}{3g_s M}\right) \tag{5.4}$$

is the warp factor at the bottom of the throat. The total potential for  $\sigma$  (with  $\alpha = 0$ ) is

$$V(\sigma) = \frac{aAe^{-a\sigma}}{2\sigma^2} \left( \frac{\sigma aA}{3} e^{-a\sigma} + W_0 + Ae^{-a\sigma} \right) + \frac{D}{4\sigma^2}. \tag{5.5}$$

For example, as shown in figure 1,  $D/A^2 = 10^{-4}$  with  $W_0/A^2 = -0.1107$ ,  $a = \pi/87$  gives a moduli potential with a meta-stable de Sitter vacuum. For validity of the geometrical description, it is important to make sure that a local minimum of the potential is at a sufficiently large value of the volume modulus  $\sigma$ . The example shown in figure 1 has a local minimum at  $\sigma \simeq 100$ , which is large enough.

We would like to make the coefficient  $D$  of  $\delta V$  sufficiently small so that the moduli potential with the non-supersymmetric correction has a local minimum at large enough  $\sigma$ . On the other hand, as we shall see in subsection 5.3, we would like to consider a large  $\bar{N}_3$  so that the anti- $D3$ -branes form a black brane. Since  $D \propto a_0^4 \bar{N}_3$ , the large  $\bar{N}_3$  threatens



**Figure 1:** The moduli potential (multiplied by  $10^9$ ) for  $D/A^2 = 10^{-4}$ ,  $W_0/A^2 = -0.1107$  and  $a = \pi/87$ . There is a meta-stable minimum at  $\sigma \simeq 100$ .

validity of the moduli stabilization while the exponential dependence (5.4) of  $a_0$  on the fluxes will certainly help reducing  $D$ .

Note that one cannot make the warp factor  $a_0$  arbitrarily small since the number of branes and fluxes must satisfy the tadpole condition

$$-\bar{N}_3 + MK = \frac{\chi}{24}, \tag{5.6}$$

where  $\chi$  is the Euler characteristics of the Calabi-Yau fourfold [40]. Ref. [45] lists examples of Calabi-Yau fourfolds in the range

$$-240 \leq \chi \leq 1820448. \tag{5.7}$$

As far as the author knows, there is no argument prohibiting Calabi-Yau fourfolds with larger  $\chi$ . However, for concreteness, we restrict our considerations to the range (5.7). The tadpole condition (5.6) combined with the inequalities (5.7) and (5.1) implies that  $K$  cannot be arbitrarily large for a fixed  $\bar{N}_3$ . This means that the warp factor given by (5.4) cannot be arbitrarily small. Therefore, in order to make  $D$  to have a small value, we must carefully choose parameters  $(\bar{N}_3, M, K, g_s)$  so that all consistency conditions are satisfied. In subsection 5.3 we shall see that this is indeed possible.

## 5.2 A tale of anti-D-branes

As already stated, anti- $D3$ -branes or other supersymmetry-breaking branes at the tip of a warped throat is one of the essential ingredients of the type IIB warped flux compactification. Without those branes, we would end up with a negative cosmological constant, which is inconsistent with observations.

The anti- $D3$ -branes are distributed over the  $S^3$  at the bottom of the warped throat, but should feel slight attractive force towards each other since supersymmetry is broken and the attractive gravitational force is not completely canceled by the repulsive R-R force. Thus, in the end, they should gather and form a bound object. The gravitational radius of the bound object made of  $\overline{N}_3$  anti- $D3$ -branes should be<sup>2</sup>

$$R_g \simeq (4\pi g_s \overline{N}_3)^{1/4} l_s. \tag{5.8}$$

It is thought that the system of anti- $D3$ -branes should relax to a non-supersymmetric NS 5-brane “giant graviton” configuration [42]. The NS 5-brane is wrapped on a  $S^2$  inside the internal  $S^3$  and carries anti- $D3$  charge  $\overline{N}_3$  coming from a world-volume magnetic flux. This configuration is classically stable for

$$\frac{\overline{N}_3}{M} < 0.08, \tag{5.9}$$

but can quantum-mechanically tunnel to a supersymmetric vacuum. The decay rate is

$$\Gamma \sim l_s^{-1} \exp\left(-\frac{27\pi^4 b_0^{12} g_s M^6}{64 \overline{N}_3^3}\right), \tag{5.10}$$

where  $b_0 \simeq 0.93266$ . Being conservative,<sup>3</sup> if we suppose  $l_s \simeq M_{Pl}^{-1}$  then  $\Gamma \ll H_0 \simeq 10^{-61} M_{Pl}$  requires that

$$\frac{g_s M^6}{\overline{N}_3^3} \gtrsim 8. \tag{5.11}$$

This condition is easily satisfied. For this reason, we consider the NS 5-brane state as a starting point of our discussion. The radius of the  $S^2$  on which the NS 5-brane is wrapped is

$$R_{S^2} = \frac{2\pi \overline{N}_3}{b_0^3 M} \sqrt{g_s M} l_s. \tag{5.12}$$

### 5.3 What does the correspondence principle require?

We shall now seek the condition under which the anti- $D3$ -branes at the tip of the warped throat should be described by a non-extremal black brane, based on the correspondence principle for  $D$ -branes and black branes [31].

Roughly speaking, the correspondence principle says that a stringy object and the corresponding black object with the same charges are different descriptions of the same object. The black object is a better description if the gravitational radius is larger than the size of the stringy object. Thus, in order for the anti- $D3$ -branes to be described by a black brane, the gravitational radius  $R_g$  given by (5.8) must be greater than both the

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<sup>2</sup>To be precise, this  $R_g$  is the gravitational radius for a BPS-saturated, isolated stack of anti- $D3$ -branes. However, (5.8) is expected to be reasonably accurate, provided that the minimum length scale  $\sqrt{g_s M} l_s$  of the throat geometry is sufficiently longer than the  $R_g$ . This condition is actually satisfied for the examples (5.14) and (5.17).

<sup>3</sup>If  $l_s$  is longer then this condition becomes weaker.



string scale  $l_s$  and the radius  $R_{S^2}$  of the non-supersymmetric NS 5-brane “giant graviton” configuration given by (5.12). Therefore, hereafter, we impose the condition

$$R_g \gtrsim R_{S^2}, \quad R_g \gtrsim l_s. \tag{5.13}$$

Under this condition, according to the correspondence principle for  $D$ -branes and black branes [31], the bound object of  $\overline{N}_3$  anti- $D3$ -branes should be described by a black brane. Note, however, that this configuration cannot be a BPS saturated state since supersymmetry is explicitly broken in the present setup. Therefore, we conclude that this must be a non-extremal black brane.

We now see that there is a range of parameters in which the condition (5.13) is compatible with the KKLT-type moduli stabilization. Again, for concreteness, we adopt the range (5.7) for the Euler characteristics of the Calabi-Yau fourfold. The goal is to find a set of parameters  $(\overline{N}_3, M, K, g_s)$  which satisfies (5.13), (5.9), (5.11), (5.1) and (5.7) simultaneously and which gives an sufficiently small value of the coefficient  $D$  of  $\delta V$ . Here,  $\chi$  should satisfy the tadpole condition (5.6) and  $D$  is given by the formula (5.3). This is indeed possible. For example,

$$\overline{N}_3 = 22, \quad M = 553, \quad K = 131, \quad g_s = 0.1 \tag{5.14}$$

satisfy all conditions, leading to

$$a_0 \simeq 7 \times 10^{-3}, \quad \frac{D}{T_3} \simeq 10^{-4}, \quad \chi = 1738104, \tag{5.15}$$

and

$$\frac{R_{S^2}^2}{R_g^2} \simeq 1, \quad \frac{l_s^2}{R_g^2} \simeq 0.19. \tag{5.16}$$

This value of  $D$  is small enough that the anti- $D3$ -brane contribution to the moduli potential is under control. Indeed, this value roughly corresponds to the value used in figure 1 if  $A^2 \simeq T_3$ . Another example is

$$\overline{N}_3 = 14, \quad M = 397, \quad K = 180, \quad g_s = 0.2, \tag{5.17}$$

which corresponds to

$$a_0 \simeq 9 \times 10^{-3}, \quad \frac{D}{T_3} \simeq 10^{-5}, \quad \chi = 1714704, \tag{5.18}$$

and

$$\frac{R_{S^2}^2}{R_g^2} \simeq 1, \quad \frac{l_s^2}{R_g^2} \simeq 0.17. \tag{5.19}$$

Note that there is no argument prohibiting  $\chi$  larger than (5.7). If Calabi-Yau fourfolds with larger  $\chi$  are found then it will be easier to satisfy all the consistency conditions.

In summary, if the number of anti- $D3$ -branes  $\overline{N}_3$ , the flux number  $M$  and the string coupling  $g_s$  satisfy the condition (5.13) then the type IIB warped flux compactification should include a non-extremal black brane at the bottom of a warped throat. As shown

explicitly, it is possible to find a set of parameters for which the condition (5.13) is compatible with the KKLT-type moduli stabilization.

Note that the gravitational radius  $R_g$  in the above examples is not large enough to make classical description of the black brane reliable. Indeed, since  $l_s^2/R_g^2 \sim 20\%$ , we should expect  $\alpha'$ -corrections of order 20% or so. We suppose that those  $\alpha'$ -corrections do not accidentally restore the symmetry broken in the classical level and that the Lorentz symmetry along the world-volume of the black brane remains spontaneously broken. In the next subsection we shall introduce another brane parallel to the world-volume of the black brane and consider it as our 4-dimensional universe. At the position of our brane the  $\alpha'$ -corrections should be negligible and the geometry can be treated classically if it is sufficiently far from the black brane.

Note also that, while we have considered stabilizing only one Kähler modulus, in general the number of Kähler moduli is not just one. Indeed, Calabi-Yau fourfolds with large Euler number, which we have considered, tend to have many Kähler moduli. As a result, rigorous treatment of Kähler moduli stabilization is rather involved. (See e.g. [46], where 51 Kähler moduli are stabilized.) In the above analysis, following KKLT and other works in the literature, we have considered only one Kähler modulus and have seen how difficult it is to achieve stabilized models. In particular, we have seen that the moduli stabilization sets strong constraints on the model parameters  $(\bar{N}_3, M, K, g_s)$  other than those directly related to the Kähler moduli stabilization  $(W_0, A, a)$ . Since inclusion of all other Kähler moduli certainly complicates the analysis, at this moment we do not know whether inclusion of them strengthens or weakens the constraints on the former set of model parameters  $(\bar{N}_3, M, K, g_s)$ . With a large number of Kähler moduli, we will certainly have extra parameters as well as extra stability conditions. Further studies towards complete analysis of moduli stabilization are certainly worthwhile pursuing.

#### 5.4 Lorentz breaking on our brane

We now introduce another brane as our universe and arrange it to be parallel to the world-volume of the black brane. Our brane may be placed either in the bulk region, where the warp factor is of order unity, or in another throat.

If the KKLT type moduli stabilization is valid then this configuration allows the induced geometry on our brane to be maximally symmetric, either Minkowski or de Sitter. Judging from what happens in the Randall-Sundrum type braneworld cosmology [47–55], one might expect that the black brane would behave as “dark radiation” [52] and that the 4-dimensional universe would expand like radiation-dominated one. However, since all moduli are stabilized, this expectation is not correct and the black brane is indistinguishable from the 4-dimensional cosmological constant from the viewpoint of the gravitational source driving the homogeneous, isotropic evolution of the 4-dimensional universe. In the Randall-Sundrum type setup without radion stabilization, the scale factor of the 4-dimensional universe on the UV brane would be the radial position of the UV brane in the 5-dimensional AdS-Schwarzschild geometry [55]. In this language the expansion of the universe would be due to the motion of the brane away from the black brane. The effect of the black brane on the UV brane would be diluted as the UV brane moves away, and

this is the reason why the effective energy density of the "dark radiation" scales as  $1/a^4$  in accord with the radial dependence of the projected Weyl tensor [56]. On the other hand, in the present setup all moduli are stabilized. Thus, the shape and the size of the internal space do not change significantly as the universe expands. Since both the black brane and our brane are located in the internal space, this means that there is no way to dilute the influence of the black brane on our 4-dimensional universe. The black brane is always there! (The Hawking radiation is negligible for tiny deviation from extremality.) Thus, we are led to the conclusion that the effective 4-dimensional energy density induced by the black brane cannot depend as strongly as the dark radiation on the scale factor of the 4-dimensional universe. In other words, the effect of the black brane on our brane is indistinguishable from a cosmological constant as far as homogeneous, isotropic evolution of the 4-dimensional universe is concerned. Therefore, the 4-dimensional universe should admit maximally symmetric solutions, either Minkowski or de Sitter spacetime.

The non-extremal black brane breaks the Lorentz symmetry along the world-volume down to the spatial rotational invariance and the spacetime translational invariance. This is due to different radial dependence of the time-time and the space-space components of the black brane metric. Intuitively, this is because the black brane has a preferred frame in which it is at rest. Since the brane representing our 4-dimensional world is parallel to the world-volume of the black brane, the Lorentz symmetry in our 4-dimensional world is spontaneously broken by the existence of the black brane but the spatial rotational invariance and the spacetime translational invariance are still preserved. Note again that the induced metric on our brane is Minkowski or de Sitter even though the Lorentz symmetry is broken. This leads to a Higgs phase of gravity in our world.

### 5.5 Horizon modes and (locally) localized gravity

Suppose that the warp factor on our brane is much larger than the small warp factor near the black brane. This is indeed the case if our brane is located in the bulk region, where the warp factor is of order unity, or in another throat shorter than the black brane throat.

In this case, due to the (locally) localized gravity phenomenon [57, 58], the overlap of the graviton zero mode with modes localized near the black brane horizon is exponentially suppressed compared with the overlap with matter on our brane. This is because the overlap is controlled by the warp factor. Therefore, gravity on our brane is essentially unaffected by modes localized near the black brane.

The black brane horizon should change the boundary condition for 10-dimensional fields near the tip of the warped throat and modify the Kaluza-Klein (KK) spectrum significantly. In a sense, this is analogous to what happens in the 't Hooft's brick wall model for black hole entropy [59, 60]. Because of the infinite blue-shift at the horizon, modes with finite frequencies measured from a distance can have arbitrarily high local frequencies near the horizon. Therefore, there can be infinite number of extra modes associated with the presence of the horizon. The 't Hooft's proposal was to relate the number of those extra modes to the black hole entropy. In the present setup, one might think that those extra modes threaten the recovery of the 4-dimensional gravity since they are expected to form a gapless mass spectrum.

Nonetheless, those extra light modes localized near the black brane do not affect gravity on our brane. We are considering a situation where the warp factor on our brane is much larger than the warp factor near the black brane. Therefore, as explained above, due to the (locally) localized gravity, those extra modes essentially do not interact with the graviton zero mode nor with matter on our brane. Thus, 4-dimensional gravity should be recovered on our brane.

The same reasoning implies that the scale of spontaneous Lorentz breaking on our brane should be rather low compared with the fundamental scale. This is because Lorentz breaking modes are localized near the black brane and, thus, has exponentially small overlap with the graviton zero mode. Despite the tininess of Lorentz breaking, the Higgs phase of gravity has dramatic consequences on gravity and cosmology in our world as reviewed in the introduction.

## 6. Summary and discussion

As a step towards a Higgs phase of gravity in string theory, we have considered a braneworld scenario with a black brane parallel to our brane. This is motivated by the following observations. (a) The structure of the low energy effective field theory (EFT) of gauged ghost condensation is determined solely by the symmetry breaking pattern. (b) Similarities between the Jeans-like instability of the Nambu-Goldstone (NG) boson and the Gregory-Laflamme (GL) instability of black branes suggest using a black brane. (c) A non-extremal black brane tends to spontaneously break the Lorentz symmetry along its world-volume. (d) In order for a black brane to break the Lorentz symmetry in our world, our world must be parallel to the world-volume of the black brane. (e) In (higher-dimensional) general relativity, branes with codimensions more than two tend to form black branes when the brane thickness is sufficiently small.

The existence of the black brane horizon spontaneously breaks the Lorentz symmetry on our brane but preserves the rotational and translational invariance. To investigate moduli stabilization, we have considered a KKLT-type moduli potential and found sets of parameters which realize stabilized moduli. We have also pointed out a possible obstacle to the use of the KKLT-type moduli potential which stabilizes only one Kähler modulus: our setup tends to include many Kähler moduli and the complete analysis should be rather involved. If the moduli stabilization remains valid after taking all Kähler moduli into account, then this setup leads to a Higgs phase of gravity in string theory and, thus, may be considered as a UV completion of the gauged ghost condensation.

If the gauge coupling in the EFT of the gauged ghost condensation is small enough then this setup reduces to the ghost condensation and the NG boson coupled to gravity exhibits Jeans-like instability. We have speculated that the geometrical counter-part of Jeans-like instability might be related to the GL instability of the non-extremal black brane.

Having proposed a possible scenario towards a UV completion of the gauged ghost condensation, let us now discuss physics beyond the low energy EFT. For example, this geometrical setup may make it possible for us to think about transition from the symmetric

phase to the broken phase. As described in subsection 5.2,  $\overline{N}_3$  anti- $D3$ -branes distributed over the  $S^3$  at the bottom of the warped throat feel attractive force towards each other and they form a bound object. Provided that  $g_s \lesssim 1$  and that  $g_s \overline{N}_3 \gtrsim 1$ , initially the gravitational radius of each anti- $D3$ -brane is less than the string scale but the gravitational radius of the final bound object is greater than the string scale. Therefore, as argued in section 5.3, while the stringy picture is a good description for the initial state, the black brane picture is more relevant for the final state. Thus, this process can be considered as gravitational collapse of a collection of anti- $D3$ -branes to form a black brane. Before the gravitational collapse, the 4-dimensional Lorentz symmetry is preserved. On the other hand, the 4-dimensional Lorentz symmetry is spontaneously broken after the formation of the black brane by gravitational collapse.

In this picture, the final state is stationary. This is the reason why the residual symmetry can include not only the 3-dimensional spatial diffeomorphism invariance but also an unbroken  $U(1)$  gauge symmetry. In the limit  $g^2 \ll g_c^2$ , the latter symmetry is translated to a global shift symmetry in the EFT of the NG boson. Therefore, it is easy to infer how we can break the shift symmetry. In the situation where  $g^2 \ll g_c^2$ , if the non-extremal black brane at the tip of the warped throat is not exactly stationary but time-dependent then the shift symmetry is broken, while the 3-dimensional spatial diffeomorphism invariance still remains. If the black brane is quasi-stationary then the breaking of the shift symmetry should be very weak.

It is certainly worthwhile considering other nonlinear and/or UV issues beyond the EFT. As an example, the UV completion might provide some new insight on nonlinear dynamics of the NG boson triggered by the Jeans-like instability for the case  $g^2 < g_c^2$ . The endpoint of the GL instability is still a question under debate. It might be a small black holes or a non-uniform black brane. In any case, according to our consideration, the endpoint of the GL instability should correspond to the endpoint of nonlinear evolution of the NG boson triggered by the Jeans-like instability. Therefore, knowledge about the endpoint of the GL instability might provide new insight on nonlinear dynamics of gravity in the Higgs phase. In particular, this line of consideration might determine the non-linear properties of an alternative to dark matter in the context of ghost condensation.

The way to end the ghost inflation is also an issue for which the UV completion might be useful. In ref. [7] it was assumed that the shift symmetry is broken locally so that an inflationary de Sitter phase ends gracefully. In the EFT language, this can, for example, be due to a phase transition in another sector triggered by the scalar field responsible for the ghost condensate. In our UV completion, as discussed above, the shift symmetry is broken if the black brane at the bottom of a warped throat is time-dependent. Thus, if the black brane experiences a transition from an almost stationary phase to another almost stationary phase then the shift symmetry is broken essentially in the transition period only. Depending on the nature of the transition, the 4-dimensional effective cosmological constant changes and the initial inflationary phase can end gracefully. The cause of the transition may be a capture of another brane by the black brane, a merger of black branes, and so on. There may be other possibilities to UV complete the idea of ghost inflation.

Couplings to the matter sector is also worthwhile investigating in the geometrical setup. In the EFT language, refs. [6, 28] listed allowed couplings between the (ungauged or gauged) ghost condensate and the matter sector, and discussed physical consequences. For example, the Lorentz- and CPT-violating Chern-Simon operator of the electromagnetic field can be forbidden by imposing a discrete symmetry [6]. It is certainly important to see if the matter sector can be embedded with this kind of discrete symmetry in the present geometrical setup.

As already stated many times, the ghost condensation can be obtained as the  $g^2 \ll g_c^2$  limit of the gauged ghost condensation, for which we have suggested a possible UV completion. In the context of the EFT, this limit can be achieved by fine-tuning the gauge coupling  $g$ . However, we do not yet know whether this limit of the geometrical setup really exists or not. It is certainly worthwhile to express actual values of the coupling constant  $g$  and the scale of symmetry breaking  $\mathcal{M}$  in terms of the parameters of the geometrical setup.

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## A. Four-parameter family of black-brane solutions

We consider the Einstein-frame metric  $ds_{10}^2$ , the dilaton  $\phi$  and the RR field  $C^{(p+1)}$  of the form

$$\begin{aligned}
 ds_{10}^2 &= -e^{2A_0(r)} dt^2 + e^{2A(r)} \delta_{ij} dx^i dx^j + e^{2B(r)} (dr^2 + r^2 \Omega_{mn}^{(8-p)} d\theta^m d\theta^n), \\
 \phi &= \phi(r), \\
 C^{(p+1)} &= -C(r) dt \prod_{i=1}^p \wedge dx^i,
 \end{aligned}
 \tag{A.1}$$

where  $i, j = 1, \dots, p$ ;  $m, n = p+2, \dots, 9$ ; and  $\Omega_{mn}^{(8-p)} d\theta^m d\theta^n$  is the metric of the unit  $(8-p)$ -sphere. The RR field  $C^{(p+1)}$  is related to the RR field strength  $F_{8-p}$  as  $F_{8-p} = e^{(3-p)\phi/2} * dC^{(p+1)}$  for  $p = 4, 5, 6$  and to the self-dual RR field strength  $F_5$  as  $F_5 = (1/\sqrt{2})(dC^{(4)} + *dC^{(4)})$  for  $p = 3$ . In order to impose the regularity of the black brane horizon, the following

expression for the Riemann tensor is useful.

$$\begin{aligned}
R^{tr}{}_{tr} &= -(A_0'' + A_0'^2 - A_0' B') e^{-2B}, \\
R^{ir}{}_{jr} &= -(A'' + A'^2 - A' B') e^{-2B} \delta_j^i, \\
R^{mr}{}_{nr} &= -\left(B'' + \frac{B'}{r}\right) e^{-2B} \delta_n^m, \\
R^{mn}{}_{m'n'} &= -B' \left(B' + \frac{2}{r}\right) e^{-2B} (\delta_{m'}^m \delta_{n'}^n - \delta_{n'}^m \delta_{m'}^n), \\
R^{ti}{}_{tj} &= -A_0' A' e^{-2B} \delta_j^i, * \\
R^{tm}{}_{tn} &= -A_0' \left(B' + \frac{1}{r}\right) e^{-2B} \delta_n^m, \\
R^{im}{}_{jn} &= -A' \left(B' + \frac{1}{r}\right) e^{-2B} \delta_j^i \delta_n^m, \\
R^{ij}{}_{i'j'} &= -A'^2 e^{-2B} (\delta_{i'}^i \delta_{j'}^j - \delta_{j'}^i \delta_{i'}^j).
\end{aligned} \tag{A.2}$$

The general solution to the type II A/B supergravity with this ansatz is obtained as a four-parameter family of  $p$ -brane solutions [61]. In the notation of [62] the four-parameter family is written as

$$\begin{aligned}
A_0(r) &= A(r) + \frac{1}{2} \ln f(r), \\
f(r) &= e^{-c_3 h(r)}, \\
A(r) &= \frac{(7-p)}{32} \left\{ -\frac{p-3}{2} c_1 + \left[ 1 + \frac{(p-3)^2}{8(7-p)} \right] c_3 \right\} h(r) \\
&\quad - \frac{7-p}{16} \ln [\cosh(k h(r)) - c_2 \sinh(k h(r))], \\
B(r) &= \frac{1}{7-p} \ln [f_-(r) f_+(r)] + \frac{p-3}{64} \left[ (p+1) c_1 + \frac{p-3}{4} c_3 \right] h(r) \\
&\quad + \frac{p+1}{16} \ln [\cosh(k h(r)) - c_2 \sinh(k h(r))], \\
\phi(r) &= \frac{7-p}{16} \left[ (p+1) c_1 + \frac{p-3}{4} c_3 \right] h(r) \\
&\quad - \frac{p-3}{4} \ln [\cosh(k h(r)) - c_2 \sinh(k h(r))], \\
C(r) &= \pm \frac{\sqrt{c_2^2 - 1} \sinh(k h(r))}{\cosh(k h(r)) - c_2 \sinh(k h(r))},
\end{aligned} \tag{A.3}$$

where

$$\begin{aligned}
h(r) &= \ln \left[ \frac{f_-(r)}{f_+(r)} \right], \\
f_{\pm}(r) &\equiv 1 \pm \left( \frac{r_0}{r} \right)^{7-p}, \\
k &= \sqrt{\frac{2(8-p)}{7-p} - c_1^2 + \frac{1}{4} \left( \frac{3-p}{2} c_1 + \frac{7-p}{8} c_3 \right)^2 - \frac{7}{16} c_3^2}.
\end{aligned} \tag{A.4}$$

The reality and regularity of the RR field in the region  $|r_0^{7-p}| \leq r^{7-p} < \infty$  require that  $k$  and  $c_2$  be real and that  $c_2 \geq 1$  and  $c_2 \leq -1$  for  $r_0^{7-d} > 0$  and  $r_0^{7-d} < 0$ , respectively. In order for  $k$  to be real,  $c_1$  and  $c_3$  must satisfy  $c_{1-} \leq c_1 \leq c_{1+}$  and  $c_3^2 \leq c_{3max}^2$ , where

$$\begin{aligned} c_{1\pm} &= -\frac{(p-3)c_3}{4(p+1)} \pm \frac{2}{p+1} \sqrt{\frac{2p(c_{3max}^2 - c_3^2)}{7-p}}, \\ c_{3max}^2 &= \frac{4(p+1)(8-p)}{p(7-p)}. \end{aligned} \tag{A.5}$$

Let us first consider the case with  $r_0^{7-d} > 0$  (we shall consider the case with  $r_0^{7-p} < 0$  later). In this case, for generic values of the parameters the Ricci scalar diverges at  $r = r_0$  ( $> 0$ ). The leading term is  $R \sim \beta_R r_0^{-2} y^{-\gamma}$ , where

$$\begin{aligned} y &= \frac{7-p}{2} \left( \frac{r}{r_0} - 1 \right), \\ \gamma &= - \left[ \frac{(p+1)(p-3)c_1}{32} + \frac{(p-3)^2 c_3}{128} + \frac{2(8-p)}{7-p} \right] + \frac{p+1}{8} k, \\ \beta_R &= 2^{-[10+4/(7-p)-(p+1)/8]} (c_2+1)^{-(p+1)/8} (7-p)^3 \times \\ &\quad \left\{ \frac{16(8-p)(p-3)^2}{(7-p)^2} - (p+1)(p^2-6p+1)c_1^2 - \frac{(p-3)(p^2-6p+1)}{2} c_1 c_3 \right. \\ &\quad \left. + \frac{(p-3)^2(p^2-14p-7)}{16(7-p)} c_3^2 + (p-3)[4(p+1)c_1 + (p-3)c_3]k \right\}. \end{aligned} \tag{A.6}$$

It is easy to show that  $\gamma$  is positive and, thus,  $R$  indeed diverges at  $r = r_0$  unless  $\beta_R = 0$ . By setting  $\beta_R = 0$ , we obtain

$$c_1 = -\frac{(p-3)c_3}{4(p+1)} - \frac{p-3}{p+1} \sqrt{\frac{p(c_{3max}^2 - c_3^2)}{2(7-p)}}. \tag{A.7}$$

With this value of  $c_1$ , the leading behavior of the Kretschmann scalar  $K \equiv R^{MN}_{M'N'} R^{M'N'}_{MN}$  near  $r = r_0$  is  $K \sim \beta_K r_0^{-4} y^{-2\gamma}$ , where

$$\begin{aligned} \beta_K &= 2^{-[10+8/(7-p)-(p+1)/4]} (c_2+1)^{-(p+1)/4} \frac{p(7-p)^3}{p+1} \\ &\quad \times \left\{ \frac{7p^3 - 74p^2 + 375p - 56}{(p+1)^2} c_3^4 - \frac{64(8-p)(p-1)}{p+1} c_3^3 \right. \\ &\quad \left. - \frac{8(p+17)(8-p)}{p+1} c_3^2 - \frac{16(8-p)(23p^3 - 386p^2 + 1319p + 2304)}{p(7-p)^2} \right. \\ &\quad \left. + 128 \left[ \frac{4(p-1)}{(p+1)^2} c_3^3 + \frac{(15-p)(8-p)}{(7-p)(p+1)} c_3^2 + \frac{4(p+9)(8-p)^2}{p(7-p)^2} \right] k \right\}, \\ k &= \frac{1}{8} \sqrt{2p(7-p) (c_{3max}^2 - c_3^2)}. \end{aligned} \tag{A.8}$$



By requiring that  $\beta_K = 0$ , we obtain<sup>4</sup>  $c_3 = -2$ . Therefore, the regularity of the horizon at  $r = r_0$  requires that

$$c_1 = -\frac{p-3}{2(7-p)}, \quad c_3 = -2 \quad \text{for } r_0^{7-d} > 0. \quad (\text{A.9})$$

The above analysis can be repeated for the case with  $r_0^{7-p} < 0$ . The leading behavior of the Ricci scalar near  $r = |r_0|$  is  $R \sim \tilde{\beta}_R |r_0|^{-2} y^{-\tilde{\gamma}}$ , where  $\tilde{\beta}_R$  and  $\tilde{\gamma}$  are  $\beta_R$  and  $\gamma$  in (A.6) with the replacement  $(c_1, c_2, c_3) \rightarrow (-c_1, -c_2, -c_3)$ . Again,  $\tilde{\gamma}$  is shown to be positive and the regularity of the Ricci scalar at  $r = |r_0|$  requires  $\tilde{\beta}_R = 0$ , which implies (A.7) with the replacement  $(c_1, c_3) \rightarrow (-c_1, -c_3)$ . With this value of  $c_1$ , the leading behavior of the Kretschmann scalar is  $K \sim \tilde{\beta}_K |r_0|^{-4} y^{-2\tilde{\gamma}}$ , where  $\tilde{\beta}_K$  is  $\beta_K$  in (A.8) with the replacement  $(c_2, c_3) \rightarrow (-c_2, -c_3)$ . Hence, by requiring  $\tilde{\beta}_K = 0$ , we obtain

$$c_1 = \frac{p-3}{2(7-p)}, \quad c_3 = 2 \quad \text{for } r_0^{7-d} < 0. \quad (\text{A.10})$$

We are now left with the two parameters  $r_0^{7-d}$  and  $c_2$ . The 10-dimensional Einstein-frame metric is

$$ds_{10}^2 = e^{2A(r)} \left[ -\left(\frac{\bar{f}_-(r)}{\bar{f}_+(r)}\right)^2 dt^2 + \delta_{ij} dx^i dx^j \right] + e^{2B(r)} (dr^2 + r^2 \Omega_{mn}^{(8-p)} d\theta^m d\theta^n), \quad (\text{A.11})$$

where

$$\begin{aligned} \bar{f}_\pm(r) &= 1 \pm \left(\frac{|r_0|}{r}\right)^{7-p}, \\ A(r) &= -\frac{7-p}{16} \ln \left[ \frac{1-|c_2|}{2} \left(\frac{\bar{f}_-(r)}{\bar{f}_+(r)}\right)^2 + \frac{1+|c_2|}{2} \right], \\ B(r) &= \frac{1}{7-p} \ln (\bar{f}_-(r) \bar{f}_+(r)) - \frac{(p-3)^2}{16(7-p)} \ln \left(\frac{\bar{f}_-(r)}{\bar{f}_+(r)}\right) \\ &\quad + \frac{p+1}{16} \ln \left[ \frac{1-|c_2|}{2} \left(\frac{\bar{f}_-(r)}{\bar{f}_+(r)}\right) + \frac{1+|c_2|}{2} \left(\frac{\bar{f}_+(r)}{\bar{f}_-(r)}\right) \right]. \end{aligned} \quad (\text{A.12})$$

The dilaton and the RR field are

$$\begin{aligned} \phi(r) &= -\frac{p-3}{4} \ln \left[ \frac{1-|c_2|}{2} \left(\frac{\bar{f}_-(r)}{\bar{f}_+(r)}\right)^2 + \frac{1+|c_2|}{2} \right], \\ C(r) &= \pm \frac{\sqrt{|c_2|^2 - 1} [\bar{f}_+(r)^2 - \bar{f}_-(r)^2]}{(1+|c_2|)\bar{f}_+(r)^2 + (1-|c_2|)\bar{f}_-(r)^2}. \end{aligned} \quad (\text{A.13})$$

Note that  $c_2 \geq 1$  for  $r_0^{7-p} > 0$  and  $c_2 \leq -1$  for  $r_0^{7-p} < 0$ . The two parameter solution actually agrees with the black  $p$ -brane solution of ref. [63] up to the coordinate transformation between the isotropic coordinate in the present analysis and the Schwarzschild-like

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<sup>4</sup>For  $p = 1$ ,  $c_3 = 2$  also satisfies  $\beta_K = 0$ . However, we need  $p \geq 3$  to accommodate the 3-dimensional spatial diffeomorphism of our 4-dimensional universe. Thus, we restrict our consideration to the cases with  $p = 3, 4, 5, 6$ . In these cases  $C_2 = -2$  is the unique solution to  $\beta_K = 0$ .

coordinate in [63]. From this form of the metric, it is seen that, for  $r_0^{7-p} \neq 0$ , the solution has a  $p$ -dimensional spatial rotational invariance but does not have a  $(p+1)$ -dimensional Lorentz symmetry. For  $r_0^{7-p} = 0$  with  $c_2$  finite, the solution is trivial.

If we take the scaling limit  $r_0^{7-p} \rightarrow \pm 0$ ,  $c_2 \rightarrow \pm\infty$  with  $c_2 r_0^{7-p}$  kept finite, the solution reduces to the BPS  $Dp$ -brane:

$$\begin{aligned} ds_{10}^2 &= f_0^{-\frac{7-p}{8}} \eta_{\mu\nu} dx^\mu dx^\nu + f_0^{\frac{p+1}{8}} (dr^2 + r^2 \Omega_{mn}^{(8-p)} d\theta^m d\theta^n), \\ e^\phi &= f_0^{-\frac{p-3}{4}}, \\ C &= \pm (1 - f_0^{-1}), \end{aligned} \tag{A.14}$$

where  $\mu, \nu = 0, 1, \dots, p$ ,  $x^0 = t$  and

$$f_0 = 1 + \frac{\mu_0}{r^{7-p}}, \quad \mu_0 \equiv 2c_2 r_0^{7-p}. \tag{A.15}$$

In this limit the  $(p+1)$ -dimensional Lorentz symmetry is restored but the extremality is also recovered at the same time.

On the other hand, for generic values of the parameters  $r_0^{7-d}$  and  $c_2$ , the solution does not possess the  $(p+1)$ -dimensional Lorentz symmetry. Therefore, we conclude that within the four-parameter family of solutions, a regular non-extremal black  $p$ -brane always breaks the  $(p+1)$ -dimensional Lorentz symmetry along its world-volume but preserves the  $p$ -dimensional spatial rotational invariance. Evidently, the  $(p+1)$ -dimensional translational invariance along the world-volume is unaffected by the non-extremality.

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